Lecture 1-2

Jan 5, 2024

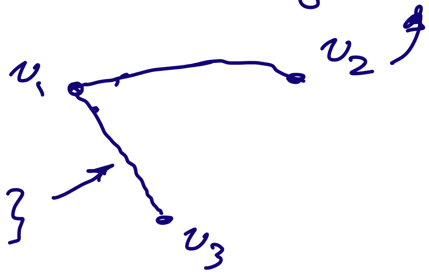
$G = (\overset{n}{V}, \overset{m}{E})$

V : vertices / Nodes

$[V]^2$ = 2-element subsets of V

$E \subseteq [V]^2$

edge set



if $\{v_1, v_3\} \subseteq E$

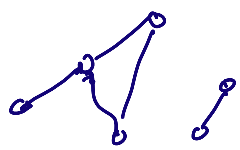
? max # of edges in graph : $\binom{n}{2} = \frac{n(n-1)}{2}$

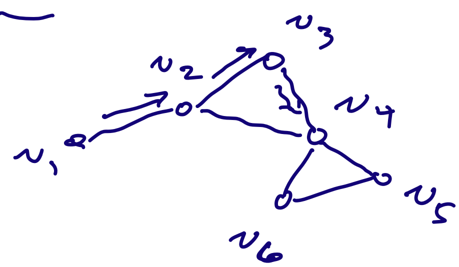
deg (v) = # of edges incident on vertex v

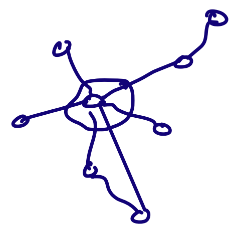
? max deg $= n-1$

connectedness:

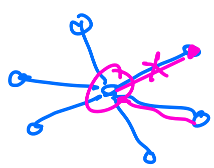
$n = 6$
$m = 5$

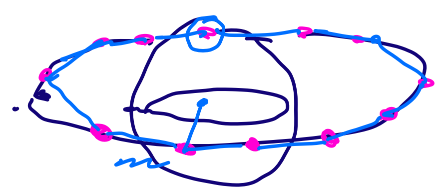


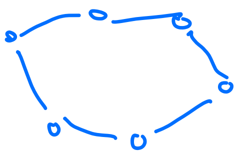




Star graph





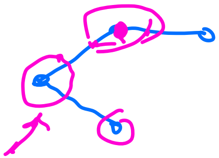


cycle

Lemma: $\sum_{v \in G} \deg(v) = 2|E| = 2m$

cardinality

$v \in V(G)$



$\deg(v) = 2$

$G \leftarrow$ directed

$E \subseteq [V]^2 \qquad \{1,2\} = \{2,1\}$

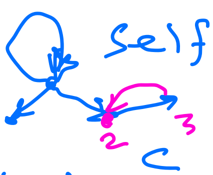
$V \times V = \{1, 2, \dots, n\} \times \{1, 2, \dots, n\}$

$= \{(1,1), (1,2), \dots, (1,n)$

$\vdots$

$(n,n)\}$

$\max |V \times V| = n^2$

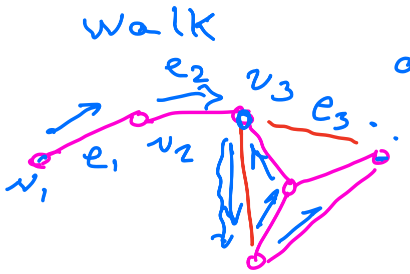


self loop

$E_{directed} \subseteq V \times V$

D = directed graph = (V, E)

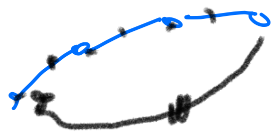
$E \subseteq V^2$

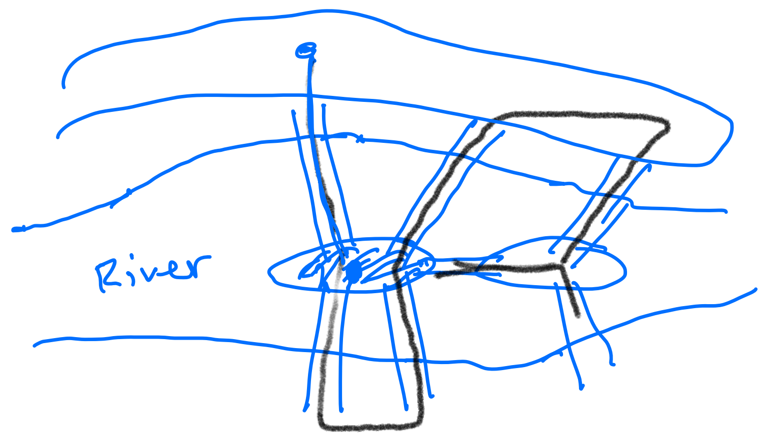


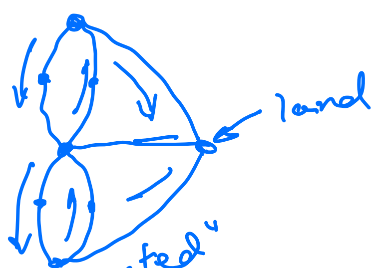
a path: a walk without vertex/node repetitions

$$W = v_1\, e_1\, v_2\, e_2 \ldots$$

Euler:



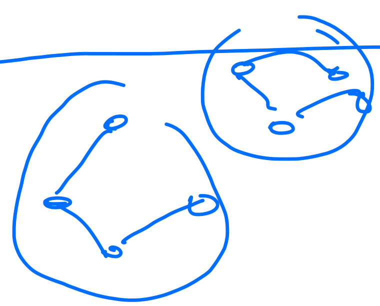




Eulerian walk

Eulerian ~~cycle~~ tour

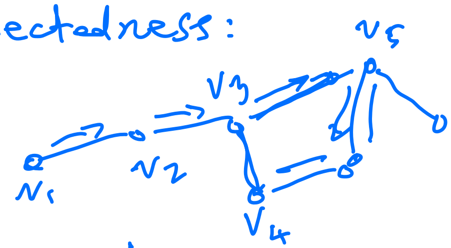
Thm: A "connected" graph G is "Eulerian" (tour) if & only if $\deg(v)$ is even $\forall v \in V$.



$O(n^2) \approx c\, n$ for some c.

2^n

? connectedness:



$\approx O(n^2)$

adjacency list

$v_1 \to$ neighbors : v_2
$v_2 \to v_3$
$v_3 \to v_4, v_5$
v_n ~~~

$O(m+n)$

Breadth First Search : BFS. / "DFS"

Input: adjacency list

Output: if $\exists$ a path from a starting vertex to another vertex

0> "explored" $\to$ alg. has touched that node.

1> a stack

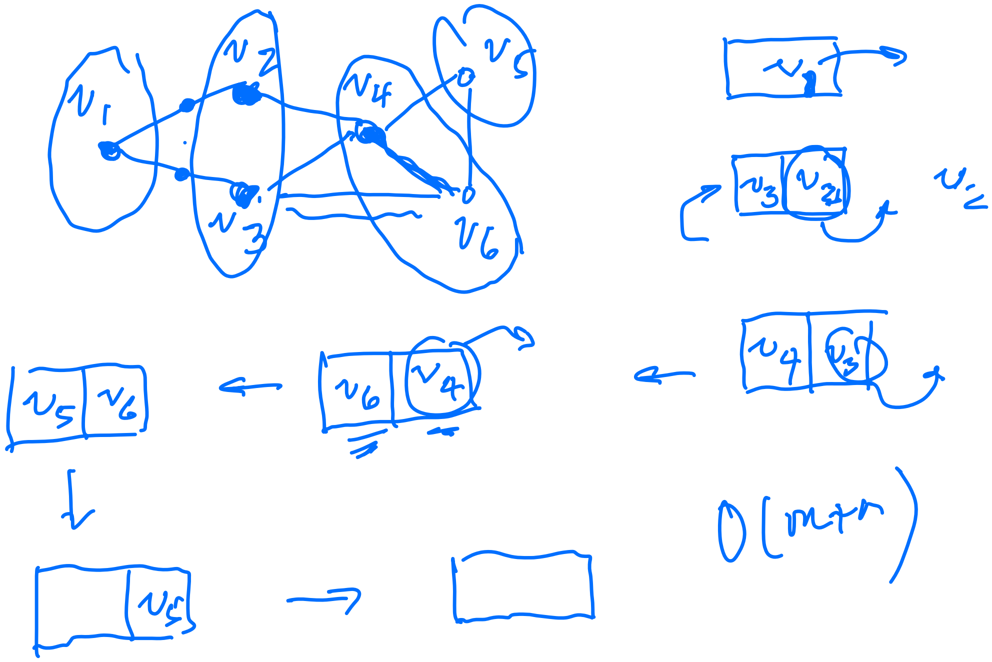
Q $\boxed{v_s}$

initialize with "v_s" as source first node of the path.

2> if Q is nonempty:

- remove v from front of Q
- for every edge (v, w) in the adjacency list of v
  - if w is unexplored the mark it as explored & add w to back of the stack.

Ex:



$O(m+n)$